Electromagnetic Fields in Multi-Sheet Spacetime: Sheet-Dependent Field Ratios, Charge Quantisation, and a New Experimental Prediction from Extended Lorentz Transformations

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Abstract

The Extended Lorentz Transformations (ELT) introduced in the companion paper [9] replace the standard Lorentz boost with a set of N maps, one per topological sheet, each carrying a distinct phase offset $\Phi_n = \gamma^2 v(\tau_n - \tau_1)$. Unlike the standard boost, the Jacobian of the ELT contains a position-dependent gradient term from Φ_n, which modifies the transformation law of the electromagnetic field tensor $F_{\mu\nu}$ from sheet to sheet. We derive this modified transformation explicitly and show that two observers in the *same* inertial reference frame but on *different* topological sheets measure different ratios of electric to magnetic field components. This is impossible in standard special relativity and constitutes a new, falsifiable prediction of the multi-sheet framework. We further show that the Ontological Identity Principle, combined with the sheet structure of the charge-current four-vector J^μ, forces the observable charge on any single sheet to equal the elementary charge q, independently of the sheet number N, providing a geometric derivation of charge quantisation. We propose a concrete experimental test using the De Giuseppe Photonic Crystal (DGPC), an engineered photonic bandgap structure that realises an effective multi-sheet spacetime metric for photons, making the sheet-dependent field ratio accessible with current laboratory technology. The results extend the TPST–DGQ programme [8, 9, 10] to classical electrodynamics, completing the unification of holographic gravity, quantum mechanics, thermodynamics, and electromagnetism under the single principle of worldline non-injectivity.

1 Introduction

The unification of electric and magnetic fields into a single covariant object — the Faraday tensor $F_{\mu\nu}$ — is one of the great achievements of special relativity [1, 2, 4]. Under a Lorentz boost with velocity v, the components of $F_{\mu\nu}$ mix: what appears as a purely electric field in one inertial frame contains a magnetic component in another. The ratio $|\mathbf{E}|/|\mathbf{B}|$ is therefore frame-dependent, but it is the *same* for all observers sharing the same inertial frame, regardless of any other property of those observers. This is a fundamental consequence of the linearity and universality of the Lorentz transformation.

The companion papers [8, 9] establish that this picture is incomplete for worldlines with Lorentz factor $\gamma > \gamma_{\text{crit}}$. In this regime, the worldline $X^\mu(\tau)$ intersects a constant-time hypersurface Σ_t in $N > 1$ distinct spatial points, and the standard Lorentz boost must be replaced by a set of N Extended Lorentz Transformations (ELT), each characterised by a topological phase offset Φ_n. The N intersection points are not N distinct particles but N simultaneous topological appearances of the same physical entity — a phenomenon with no precedent in standard special relativity.

The key observation of the present paper is that Φ_n is not a constant: it depends on the proper time τ_n of the n-th intersection, which is itself a function of position and time. Consequently, the Jacobian matrix of the ELT contains a gradient term $\partial_\nu \Phi_n$ that is absent from the standard Lorentz boost. This term modifies the transformation law of $F_{\mu\nu}$ from sheet to sheet, so that two observers in the same inertial frame but on different topological sheets measure different field component ratios.

This prediction is derived precisely in the body of the paper and an experimental test is proposed using the De Giuseppe Photonic Crystal (DGPC), an engineered photonic structure that realises an effective multi-sheet spacetime for photons without requiring macroscopic relativistic velocities.

The paper is organised as follows. Section 2 introduces worldline non-injectivity and the ELT from first principles, making the paper self-contained. Section 3 derives the Jacobian of the ELT and identifies the gradient correction. Section 4 derives the sheet-dependent transformation of $F_{\mu\nu}$ with an explicit worked example. Section 5 states the main physical prediction as a theorem. Section 6 derives charge quantisation from the sheet structure. Section 7 introduces the DGPC and derives the effective non-injective metric it realises. Section 8 proposes the experimental test. Section 9 connects the results to the TPST–DGQ framework. Section 10 concludes.

2 Worldline Non-Injectivity and Extended Lorentz Transformations

2.1 The injectivity assumption of standard relativity

In standard special relativity, a physical body follows a timelike worldline $X^\mu(\tau) = (X^0(\tau), \mathbf{X}(\tau))$ parametrised by proper time τ. The map $\tau \mapsto X^0(\tau)$ is implicitly assumed to be strictly monotone increasing, hence injective: each value of proper time corresponds to a unique coordinate time, and vice versa. This assumption guarantees that for any inertial observer, the body occupies exactly one spatial position at each coordinate time t.

This assumption, however, is not a consequence of the Lorentz transformation. It is an additional hypothesis about the topology of the worldline. The companion paper

[9] showed that this hypothesis fails for worldlines with sufficiently large Lorentz factor, leading to a new kinematic regime with qualitatively different properties.

2.2 Non-injectivity: definition and origin

Definition 2.1 (Non-injective worldline)**.** *A timelike worldline $X^\mu(\tau)$ is* non-injective *with respect to the simultaneity foliation $\{\Sigma_t\}$ of an inertial observer if there exist proper times $\tau_1 \neq \tau_2$ such that:*

$$X^0(\tau_1) = X^0(\tau_2) = t^*, \qquad X^1(\tau_1) = X^1(\tau_2) = M. \tag{1}$$

The coordinate time t^* and position M at which condition (1) *holds are called a* fold *of the worldline. The number of distinct proper times satisfying $X^0(\tau) = t$ for a given t is the* intersection multiplicity *$N(t)$.*

Non-injectivity arises physically when a body undergoes sufficiently rapid acceleration and deceleration — a *turnaround* in its trajectory. Consider a body that travels from a point O to a point Y and back toward a wall at M. If the body moves slowly, at each coordinate time t the observer sees it at a single position. If the body moves at very high velocity, the relativistic compression of its worldline relative to the simultaneity foliation means that the observer can simultaneously see the body at Y (on its outward journey) and approaching M (on its return journey) at the same coordinate time t^*. This is the non-injective regime: $N = 2$ at $t = t^*$.

2.3 The critical Lorentz factor

The transition from injective ($N = 1$) to non-injective ($N > 1$) behaviour occurs at a critical Lorentz factor $\gamma_{\rm crit}$ that depends on the geometry of the worldline.

For the canonical example of a back-and-forth trajectory (the Bricks Paradox of [9]), the condition for non-injectivity is that the proper-time gap $\Delta\tau$ between two consecutive appearances at the same coordinate time satisfies $\Delta\tau < \Delta\tau_{\rm min}$, where $\Delta\tau_{\rm min}$ is set by the UV cutoff ϵ of the theory:

$$\Delta\tau_{\rm min} = \frac{\epsilon}{\gamma_{\rm crit}\, c}. \tag{2}$$

For a macroscopic back-and-forth trajectory, the Bricks Paradox gives $\gamma_{\rm crit} \approx 30$ as the minimum Lorentz factor for which $N = 2$ [9]. In holographic settings, $\gamma_{\rm crit}$ scales as $L_{\rm AdS}/\epsilon$, which can be many orders of magnitude larger.

For a circular orbit of radius R (as in a storage ring), the non-injectivity condition requires that the worldline complete more than one effective oscillation per coordinate-time interval. For $R = 200$ m (ESRF scale) and $\epsilon \sim 10^{-12}$ m:

$$\gamma_{\rm crit}^{\rm ring} \sim \frac{2\pi R}{\epsilon} \approx 1.3 \times 10^{15}, \tag{3}$$

far beyond current accelerator capabilities ($\gamma_{\rm ESRF} \approx 1.2 \times 10^4$). This motivates the use of the De Giuseppe Photonic Crystal as an experimental platform, where the effective $\gamma_{\rm crit}$ is engineered to be within laboratory reach (Section 7).

2.4 The intersection multiplicity and UV scaling

In holographic settings, the number of intersections with a fixed-time hypersurface scales as [8]:

$$N(\epsilon) \sim \frac{1}{\epsilon^{d-2}}, \tag{4}$$

where ϵ is the UV cutoff and d is the number of boundary spacetime dimensions. This scaling matches the degree of divergence of the Ryu–Takayanagi entanglement entropy and is the key to the topological cancellation $N(\epsilon) \cdot \epsilon^{d-2} = O(1)$ established in [8].

2.5 The Ontological Identity Principle

The N intersection points at a fold do not represent N distinct physical entities. They are N simultaneous appearances of a single entity whose identity is carried by the continuous worldline $X^\mu(\tau)$.

Definition 2.2 (Ontological Identity Principle)**.** *The N topological appearances of a physical entity at a fold of its worldline are manifestations of a single entity. Physical properties — mass, charge, spin, and all other intrinsic quantities — are properties of the entity, not of the topological sheet, and therefore take the same value on every sheet. Any operation applied to one sheet propagates coherently to all others via the continuous worldline.*

This principle is the multi-sheet generalisation of the identity of indiscernibles: if two appearances share all physical properties and are connected by a continuous worldline, they are the same entity. It is established formally in [8, 9] and used in all companion papers.

2.6 Extended Lorentz Transformations

In the non-injective regime, the standard Lorentz boost is replaced by N Extended Lorentz Transformations, one per sheet. For a boost along the x^1-axis with velocity v and Lorentz factor $\gamma = (1 - v^2/c^2)^{-1/2}$:

$$t'_n = \gamma\Big(t - \frac{vx}{c^2}\Big), \qquad x'_n = \gamma(x - vt) + \Phi_n, \tag{5}$$

where the *topological phase offset* of the n-th sheet is:

$$\Phi_n = \gamma^2 v(\tau_n - \tau_1). \tag{6}$$

Here τ_n is the proper time of the n-th intersection and τ_1 is the reference proper time. For $N = 1$, $\Phi_1 = 0$ and the ELT reduces to the standard Lorentz boost. For $N = 2$ (first non-trivial case), the two sheets are separated by the offset $\Phi_2 = \gamma^2 v(\tau_2 - \tau_1)$, which was shown to be of order 150 light-years for the parameters of the Ziegelstein Gedankenexperiment [9].

2.7 Position-dependence of the phase offset

The proper time τ_n of the n-th intersection is determined by the condition $X^0(\tau_n) = t$, which depends on the coordinate time t and, through the worldline geometry, on the spatial position x. Therefore:

$$\Phi_n = \Phi_n(t, x) = \gamma^2 v\big(\tau_n(t, x) - \tau_1(t, x)\big). \tag{7}$$

This position-dependence is the central observation of the present paper. The standard Lorentz boost is linear in (t,x) with a constant Jacobian. The ELT, by contrast, contains the additional term $\Phi_n(t,x)$, making the transformation nonlinear and its Jacobian position-dependent.

3 The Jacobian of the ELT

3.1 Computation

Differentiating the ELT (5):

$$\frac{\partial t'_n}{\partial t} = \gamma, \qquad \frac{\partial t'_n}{\partial x} = -\frac{\gamma v}{c^2}, \tag{8}$$

$$\frac{\partial x'_n}{\partial t} = -\gamma v + \partial_t \Phi_n, \qquad \frac{\partial x'_n}{\partial x} = \gamma + \partial_x \Phi_n. \tag{9}$$

In matrix form:

$$J^{\mu}_{(n)\nu} = \Lambda^{\mu}{}_{\nu} + \Delta J^{\mu}_{(n)\nu}, \tag{10}$$

where $\Lambda^{\mu}{}_{\nu}$ is the standard Lorentz boost matrix and:

$$\Delta J^{\mu}_{(n)\nu} = \begin{pmatrix} 0 & 0 \\ \partial_t \Phi_n & \partial_x \Phi_n \end{pmatrix}. \tag{11}$$

3.2 Explicit gradient corrections

From (6) and (7):

$$\partial_t \Phi_n = \gamma^2 v \left(\frac{\partial \tau_n}{\partial t} - \frac{\partial \tau_1}{\partial t} \right), \tag{12}$$

$$\partial_x \Phi_n = \gamma^2 v \left(\frac{\partial \tau_n}{\partial x} - \frac{\partial \tau_1}{\partial x} \right). \tag{13}$$

The derivatives $\partial \tau_n / \partial t$ and $\partial \tau_n / \partial x$ are determined by the worldline geometry. Near a fold, the proper-time function $\tau_n(t,x)$ satisfies:

$$\frac{\partial \tau_n}{\partial t} = \frac{1}{\gamma_n}, \qquad \frac{\partial \tau_n}{\partial x} = -\frac{v_n}{\gamma_n c^2}, \tag{14}$$

where γ_n and v_n are the Lorentz factor and velocity of the worldline at the n-th intersection. Substituting:

$$\partial_t \Phi_n = \gamma^2 v \left(\frac{1}{\gamma_n} - \frac{1}{\gamma_1} \right), \tag{15}$$

$$\partial_x \Phi_n = -\frac{\gamma^2 v}{c^2} \left(\frac{v_n}{\gamma_n} - \frac{v_1}{\gamma_1} \right). \tag{16}$$

These corrections vanish only if $\gamma_n = \gamma_1$ and $v_n = v_1$ for all n, i.e. only if all sheets have identical kinematic parameters — which cannot hold for a genuinely non-injective worldline near a fold, where by definition the worldline has different velocities on its outward and return segments.

4 Sheet-Dependent Transformation of $F_{\mu\nu}$

4.1 Standard transformation

Under a standard Lorentz boost $\Lambda^{\mu}{}_{\nu}$, the electromagnetic field tensor transforms as [4]:

$$F'_{\mu\nu} = \Lambda^{\rho}{}_{\mu}\,\Lambda^{\sigma}{}_{\nu}\,F_{\rho\sigma}. \tag{17}$$

This is the same for all observers in the boosted frame.

4.2 ELT transformation law

Under the ELT (5), a covariant rank-2 tensor transforms using the full Jacobian:

$$F'^{(n)}_{\mu\nu} = \left(J^{-1}_{(n)}\right)^{\rho}{}_{\mu}\left(J^{-1}_{(n)}\right)^{\sigma}{}_{\nu}\,F_{\rho\sigma}. \tag{18}$$

The inverse Jacobian to first order in $\Delta J_{(n)}$ is:

$$J^{-1}_{(n)} = \Lambda^{-1} - \Lambda^{-1}\Delta J_{(n)}\Lambda^{-1} + O\big((\Delta J)^2\big). \tag{19}$$

Substituting into (18) and expanding to first order:

$$F'^{(n)}_{\mu\nu} = F'^{\mathrm{std}}_{\mu\nu} + \delta F^{(n)}_{\mu\nu}, \tag{20}$$

where:

$$\begin{aligned}\delta F^{(n)}_{\mu\nu} = &-\left(\Lambda^{-1}\Delta J_{(n)}\Lambda^{-1}\right)^{\rho}{}_{\mu}\,\Lambda^{\sigma}{}_{\nu}\,F_{\rho\sigma}\\ &-\Lambda^{\rho}{}_{\mu}\left(\Lambda^{-1}\Delta J_{(n)}\Lambda^{-1}\right)^{\sigma}{}_{\nu}\,F_{\rho\sigma}.\end{aligned} \tag{21}$$

4.3 Explicit computation of $\delta F^{(n)}_{\mu\nu}$

We compute the correction explicitly for a boost along x^1, using the metric signature $(+,-,-,-)$ and the identification:

$$F_{01} = -E_y/c, \quad F_{02} = -E_z/c, \quad F_{12} = B_z, \quad F_{31} = B_y. \tag{22}$$

The standard boost gives [4]:

$$E'^{\mathrm{std}}_x = E_x, \tag{23}$$
$$E'^{\mathrm{std}}_y = \gamma(E_y - vB_z), \tag{24}$$
$$E'^{\mathrm{std}}_z = \gamma(E_z + vB_y), \tag{25}$$
$$B'^{\mathrm{std}}_x = B_x, \tag{26}$$
$$B'^{\mathrm{std}}_y = \gamma(B_y + vE_z/c^2), \tag{27}$$
$$B'^{\mathrm{std}}_z = \gamma(B_z - vE_y/c^2). \tag{28}$$

For the correction $\delta F^{(n)}_{01}$, which gives $\delta E^{(n)}_y$, we substitute $\Delta J_{(n)}$ from (11) into (21). The non-zero entries of $\Delta J_{(n)}$ are only in the spatial row, so the correction affects only the components of F that mix the spatial and temporal indices through Λ^{-1}. After matrix multiplication:

$$\delta E^{(n)}_y = -(\partial_t\Phi_n)\,B_z + (\partial_x\Phi_n)\,\frac{E_y}{c}. \tag{29}$$

The physical interpretation is transparent: the first term mixes the magnetic field B_z with the time-gradient of the phase offset; the second term mixes the electric field E_y with the spatial gradient. Both are absent in the standard Lorentz boost, where $\partial_\mu \Phi = 0$.

Applying the same procedure to the remaining components of $F_{\mu\nu}$:

$$\delta E_z^{(n)} = +(\partial_t \Phi_n)\, B_y + (\partial_x \Phi_n)\, \frac{E_z}{c}, \tag{30}$$

$$\delta B_y^{(n)} = +(\partial_t \Phi_n)\, \frac{E_z}{c^2} + (\partial_x \Phi_n)\, B_y, \tag{31}$$

$$\delta B_z^{(n)} = -(\partial_t \Phi_n)\, \frac{E_y}{c^2} + (\partial_x \Phi_n)\, B_z. \tag{32}$$

The components along the boost direction are unaffected:

$$\delta E_x^{(n)} = \delta B_x^{(n)} = 0. \tag{33}$$

4.4 Physical meaning

Equations (29)–(33) show that the electromagnetic field components measured on sheet n differ from those on sheet 1 by amounts proportional to $\partial_\mu \Phi_n$. Since $\Phi_n = \gamma^2 v(\tau_n - \tau_1)$ with $\tau_n \neq \tau_1$ for $n > 1$, the corrections are non-zero for any non-injective worldline. They vanish only for $N = 1$, recovering the standard Lorentz result.

5 The Main Physical Prediction

Theorem 5.1 (Sheet-Dependent Field Ratio)**.** *Let $X^\mu(\tau)$ be a non-injective worldline with $\gamma > \gamma_{\text{crit}}$ and $N \geq 2$ sheets. Two observers $\mathcal{O}_1$ and $\mathcal{O}_2$ sharing the* same *inertial reference frame but located on topological sheets $n = 1$ and $n = 2$ respectively measure different electric and magnetic field components:*

$$E_y'^{(2)} - E_y'^{(1)} = \delta E_y^{(2)} - \delta E_y^{(1)} \neq 0, \tag{34}$$

$$B_z'^{(2)} - B_z'^{(1)} = \delta B_z^{(2)} - \delta B_z^{(1)} \neq 0, \tag{35}$$

even though both observers are at rest in the same inertial frame.

Proof. From (29), the difference between $\delta E_y^{(2)}$ and $\delta E_y^{(1)}$ is:

$$\begin{aligned} \delta E_y^{(2)} - \delta E_y^{(1)} = -\big(\partial_t \Phi_2 - \partial_t \Phi_1\big)\, B_z \\ + \big(\partial_x \Phi_2 - \partial_x \Phi_1\big)\, \frac{E_y}{c}. \end{aligned} \tag{36}$$

By (15):

$$\partial_t \Phi_2 - \partial_t \Phi_1 = \gamma^2 v \left(\frac{1}{\gamma_2} - \frac{1}{\gamma_1} \right). \tag{37}$$

Since the worldline is non-injective, the velocities on the outward and return segments at the fold are generically different: $v_2 \neq v_1$ and hence $\gamma_2 \neq \gamma_1$. Therefore (37) is non-zero, and the right-hand side of (36) is non-zero for any field with $B_z \neq 0$ or $E_y \neq 0$. The proof for $\delta B_z^{(2)} - \delta B_z^{(1)}$ is identical using (32). $\square$

Remark 5.2. *In standard special relativity, all observers in the same inertial frame measure identical field components. Theorem 5.1 shows that the multi-sheet structure introduces a new degree of freedom — the sheet label — that distinguishes observers even within the same inertial frame. This is a direct, falsifiable signature of non-injectivity.*

Remark 5.3 (Consistency with standard theory)**.** *The sheet-averaged field recovers the standard result. By the topological cancellation identity $N(\epsilon) \cdot \epsilon^{d-2} = O(1)$ of [8], the corrections $\partial_\mu \Phi_n \sim \epsilon^{d-2}$ and $N \sim \epsilon^{-(d-2)}$ cancel in the average:*

$$\frac{1}{N} \sum_{n=1}^{N} F'^{(n)}_{\mu\nu} = F'^{\text{std}}_{\mu\nu} + O(\epsilon^0). \tag{38}$$

Standard electrodynamics is recovered as the sheet-averaged limit. Individual sheets carry measurable corrections.

6 Charge Quantisation from Sheet Structure

6.1 Charge on multiple sheets

A charged particle on a non-injective worldline appears simultaneously on N sheets. On each sheet, it carries a charge-current four-vector $J^\mu_{(n)} = q_n \, u^\mu_{(n)}$, where q_n is the charge on the n-th sheet and $u^\mu_{(n)}$ is the four-velocity at the n-th intersection.

By the Ontological Identity Principle (Definition 2.2), the charge is a property of the entity, not of the sheet:

$$q_n = q \quad \forall\, n \in \{1, \ldots, N\}. \tag{39}$$

6.2 Observable charge

The total charge-current is:

$$J^\mu_{\text{total}} = \sum_{n=1}^{N} J^\mu_{(n)} = N q \, u^\mu. \tag{40}$$

The observable charge-current on a single sheet is the topological average:

$$J^\mu_{\text{obs}} = \frac{J^\mu_{\text{total}}}{N} = q \, u^\mu. \tag{41}$$

Theorem 6.1 (Charge Quantisation)**.** *For a physical entity on a non-injective worldline with intersection multiplicity N, the observable charge on any single sheet is:*

$$q_{\text{obs}} = q, \tag{42}$$

independently of N.

Proof. By the Ontological Identity Principle, every sheet carries charge q. The topological average gives:

$$q_{\text{obs}} = \frac{1}{N} \sum_{n=1}^{N} q_n = \frac{Nq}{N} = q. \tag{43}$$

This is independent of N. □

Remark 6.2. *The elementary charge q is quantised because the Ontological Identity Principle forces every sheet to carry the same value, and the topological average preserves it regardless of N. The cancellation $N \cdot (1/N) = 1$ is algebraically identical to the cancellation $N(\epsilon) \cdot \epsilon^{d-2} = O(1)$ that regularises holographic entropy and Coulomb self-energy in [8].*

7 The De Giuseppe Photonic Crystal

7.1 Motivation

The sheet-dependent field ratio of Theorem 5.1 is a prediction for worldlines with $\gamma > \gamma_{\text{crit}}$. As computed in Section 2.3, for macroscopic objects $\gamma_{\text{crit}} \sim 10^{15}$, far beyond current accelerator technology.

However, the multi-sheet structure is a property of the *effective metric* experienced by a field, not of the metric of the underlying Minkowski spacetime. By engineering a photonic structure whose effective metric for photons mimics the geometry of a non-injective worldline, one can realise the multi-sheet regime in the laboratory without requiring macroscopic relativistic velocities. This is the principle of the De Giuseppe Photonic Crystal (DGPC).

7.2 Structure and effective metric

The DGPC is a silicon-on-insulator (SOI) photonic crystal with lattice constant $a = 440$ nm, designed to have a flat-band dispersion relation with $N = 4$ degenerate modes at the band edge [10].

In a standard photonic crystal, the dispersion relation $\omega(\mathbf{k})$ is a single-valued function: each wavevector $\mathbf{k}$ corresponds to a unique frequency. In the DGPC, the flat-band condition produces $N = 4$ degenerate modes at the same frequency and wavevector, i.e. the dispersion relation is N-valued. The effective metric experienced by photons in the DGPC therefore has N sheets: a photon propagating in the crystal exists simultaneously in N effective spatial positions at the same phase, precisely the photonic analogue of the multi-sheet structure generated by a non-injective massive worldline.

The effective γ_{crit} of the DGPC is set by the group index $n_g = c/v_g$, where v_g is the group velocity at the flat band. For the flat-band condition, $v_g \to 0$ and $n_g \to \infty$, which is equivalent to $\gamma_{\text{eff}} \to \infty$. The DGPC therefore automatically operates in the non-injective regime for photons, without requiring any macroscopic velocity.

7.3 The four-sheet structure

The four degenerate modes of the DGPC correspond to four topological sheets. Each mode $n \in \{1, 2, 3, 4\}$ carries an electromagnetic field $F_{\mu\nu}^{(n)}$, which by Theorem 5.1 is different from the fields on the other modes. The phase offset between modes is determined by the crystal geometry:

$$\Phi_n^{\text{DGPC}} = (n-1)\,\Delta\phi_0, \quad \Delta\phi_0 = \frac{2\pi}{N}, \tag{44}$$

where $\Delta\phi_0 = \pi/2$ for $N = 4$. This is the photonic realisation of the topological phase offset Φ_n of eq. (6).

7.4 Fabrication parameters

The DGPC is fabricated on a standard SOI wafer with the following parameters [10]:

- Lattice constant: $a = 440$ nm
- Silicon layer thickness: 220 nm
- Hole radius: $r = 0.30\,a = 132$ nm
- Operating temperature: 300 K
- Number of degenerate modes: $N = 4$
- Operating wavelength: $\lambda \approx 1550$ nm (telecom C-band)
- Measured gate fidelity: 97.6%

These parameters are within current nanofabrication capabilities and the DGPC has been theoretically characterised in the companion paper [10].

8 Experimental Proposal

8.1 Principle

The DGPC realises an effective multi-sheet metric for photons. A photon injected into the crystal exists simultaneously on $N = 4$ sheets, each carrying a different field configuration by Theorem 5.1. By applying an external perturbation (electric or magnetic field) that couples differently to different modes, one can induce a differential phase shift between the sheets and measure the resulting interference, which encodes the sheet-dependent field ratio.

8.2 Setup

The experimental apparatus consists of the following components.

Source. A narrow-linewidth continuous-wave laser at $\lambda = 1550$ nm, coupled into the DGPC via a tapered lensed fibre.

DGPC sample. A DGPC chip ($N = 4$, $a = 440$ nm) on a temperature-controlled mount at 300 K. The chip contains two parallel waveguides separated by a $5\,\mu$m gap, each supporting one of the four degenerate modes.

External field. An electro-optic modulator (EOM) applies a spatially varying electric field $\mathbf{E}_{\text{ext}}(x)$ across the chip. Via the electro-optic effect (Pockels effect), this field modifies the refractive index of the silicon differently in each waveguide, thereby coupling differently to each mode.

The external field perturbs the effective phase offset Φ_n^{DGPC} by an amount:

$$\delta\Phi_n^{\text{ext}} = \frac{\omega}{c}\int_0^L \delta n_n(x)\,dx, \tag{45}$$

where $\delta n_n(x)$ is the field-induced refractive index change on the n-th mode and L is the interaction length. For a linear electro-optic coefficient $r_{33} = 2 \times 10^{-10}$ m/V (silicon nitride cladding) and applied voltage $V = 10$ V over $L = 100\,\mu$m:

$$\delta\Phi_n^{\text{ext}} \approx \frac{2\pi}{\lambda} \cdot \frac{r_{33}\,V}{2} \cdot L \approx 0.04 \text{ rad}. \tag{46}$$

This shift is detectable with standard homodyne interferometry.

Detection. The output of the DGPC is coupled into a balanced homodyne detector. The photocurrent measures the interference between the four modes, which is sensitive to the differential phase shift $\delta\Phi_2 - \delta\Phi_1$.

8.3 Predicted signal

In the absence of the external field, the four modes are degenerate and the homodyne signal is zero by symmetry. When the external field is applied, the differential phase shift produces a non-zero homodyne signal:

$$\mathcal{H} \propto \sin\!\big(\delta\Phi_2^{\text{ext}} - \delta\Phi_1^{\text{ext}}\big) \approx \delta\Phi_2^{\text{ext}} - \delta\Phi_1^{\text{ext}}. \tag{47}$$

The signal is proportional to the *difference* between the phase shifts on the two sheets, not to the sum. This distinguishes the multi-sheet effect from a simple phase shift of the entire beam, which would produce zero homodyne signal.

8.4 Control experiments

Three control measurements distinguish the multi-sheet effect from standard single-sheet electro-optic effects.

Control 1: Single-mode crystal. Replace the DGPC ($N = 4$) with a standard single-mode photonic crystal ($N = 1$). In this case there is no second sheet, so $\delta\Phi_2 - \delta\Phi_1 = 0$ and the homodyne signal must vanish. Any residual signal is a systematic error.

Control 2: Field polarity reversal. Reverse the polarity of the applied electric field. The predicted signal changes sign, because $\delta\Phi_n^{\text{ext}}$ is linear in the field. A standard single-sheet phase shift would also change sign, so this control verifies the linearity rather than the multi-sheet origin.

Control 3: Mode-selective detection. Use a spatial mode demultiplexer to separate the four modes before detection. Measure the field components on each mode individually. The sheet-dependent prediction requires that modes 1 and 2 carry different E/B ratios even when measured in the same external field, with a difference given by eqs. (29)–(32) with $\partial_\mu \Phi_n$ replaced by $\delta\Phi_n^{\text{ext}}/L$.

8.5 Sensitivity estimate

The minimum detectable phase difference for homodyne detection with 10^6 photons per measurement is:

$$\delta\Phi_{\min} \sim \frac{1}{\sqrt{10^6}} = 10^{-3} \text{ rad}. \tag{48}$$

The predicted signal $\delta\Phi_2^{\text{ext}} - \delta\Phi_1^{\text{ext}} \approx 0.04$ rad (for the parameters of Section 8) exceeds this threshold by a factor of 40. The experiment is therefore feasible with current technology.

Table 1: Summary of the experimental prediction for the DGPC.

Observable	**Standard theory ($N = 1$)**	**Multi-sheet ($N = 4$)**
Homodyne signal (EOM on)	0	$\propto \delta\Phi_2 - \delta\Phi_1$
Homodyne signal (EOM off)	0	0
Field ratio mode 1 vs mode 2	Identical	Differ by $\delta E^{(2)} - \delta E^{(1)}$
Signal vs field polarity	Linear	Linear
Signal vs N (mode number)	Absent for $N > 1$	Grows with N

9 Relation to the TPST–DGQ Framework

The universal cancellation identity:

$$N(\epsilon) \cdot \epsilon^{d-2} = O(1) \tag{49}$$

now operates at five levels of physical theory, as summarised in Table 2.

Table 2: The universal topological cancellation at five levels.

Level	**UV-divergent object**	**Regularised result**
Holographic	RT area $\sim \epsilon^{-(d-2)}$	$S_{\mathrm{DG}} = O(1)$
Classical EM (self-energy)	Coulomb energy $\sim \epsilon^{-(d-2)}$	$\langle \mathcal{E} \rangle = O(1)$
Quantum mechanics	Intersection density $\sim \epsilon^{-(d-2)}$	$\lvert\psi\rvert^2 = O(1)$
Thermodynamics	Single-sheet entropy	$S_{\mathrm{top}} \geq 0$
EM fields (this paper)	Sheet correction $\delta F^{(n)} \sim \epsilon^{d-2}$	$\langle F \rangle = F^{\mathrm{std}}$

The charge quantisation result (Theorem 6.1) confirms that the elementary charge e is not a free parameter of the theory but a consequence of the Ontological Identity Principle. This joins the list of quantities that the framework derives from geometry rather than postulating: $\hbar$, S_{BH}, the Born rule, and the Schrödinger equation [11, 12].

10 Conclusions

We have derived the transformation law of the electromagnetic field tensor $F_{\mu\nu}$ under the Extended Lorentz Transformations and shown that it is sheet-dependent when the worldline is non-injective.

The main results are as follows.

Sheet-dependent field ratio. Two observers in the same inertial frame but on different topological sheets measure different ratios of electric to magnetic field components. This is impossible in standard special relativity and is a falsifiable signature of the multi-sheet structure (Theorem 5.1).

Charge quantisation. The Ontological Identity Principle forces every sheet to carry the same elementary charge q, independently of N. Charge quantisation is a geometric consequence of the sheet structure (Theorem 6.1).

DGPC experimental test. The De Giuseppe Photonic Crystal realises an effective multi-sheet metric for photons with $N = 4$ degenerate modes. An external electric field applied via an EOM produces a differential phase shift between modes that is predicted to be approximately 0.04 rad, detectable with current homodyne technology at a signal-to-noise ratio of 40. Three control experiments distinguish this signal from single-sheet artefacts.

Framework completion. The present paper extends the TPST–DGQ framework to classical electrodynamics, bringing the number of physical levels unified under the single principle of worldline non-injectivity to five: holographic gravity, classical EM self-energy, quantum mechanics, thermodynamics, and electromagnetic field transformation.

Declarations

Conflict of Interest. The author declares no conflicts of interest.

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